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Leptogenesis, Gravitino Dark Matter and Entropy Production

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Abstract

Many extensions of the Standard Model predict super-weakly interacting particles, which typically have to decay before Big Bang Nucleosynthesis (BBN). The entropy produced in the decays may help to reconcile thermal leptogenesis and BBN in scenarios with gravitino dark matter, which is usually difficult due to late decays of the next-to-lightest supersymmetric particle (NLSP) spoiling the predictions of BBN. We study this possibility for a general neutralino NLSP.

We elaborate general properties of the scenario and strong constraints on the entropy-producing particle. As an example, we consider the saxion from the axion multiplet and show that, while enabling a solution of the strong CP problem, it can also produce a suitable amount of entropy.

1 Introduction

Thermal leptogenesis [1] is an attractive mechanism for generating the baryon asymmetry of the Universe, since it requires no additional ingredients beyond the see-saw scenario [2, 3, 4, 5, 6] introduced to explain the smallness of the observed neutrino masses. However, in supergravity theories the required large reheating temperature results in a copious production of gravitinos [7, 8]. If the gravitino is heavier than other superparticles, this is problematic since it typically decays during or after Big Bang Nucleosynthesis (BBN) due to its extremely weak interactions. The energetic decay products can then cause unacceptable changes of the primordial light element abundances [9, 10]. If on the other hand the gravitino is the lightest supersymmetric particle (LSP), high temperatures tend to lead to a relic gravitino density exceeding the observed dark matter density [11]. In addition, the next-to-lightest supersymmetric particle (NLSP) becomes long-lived and thus can cause similar problems with BBN as an unstable gravitino [12].

Apart from abandoning supersymmetry or thermal leptogenesis, the gravitino problem has two further solutions. The first one is to reconcile thermal leptogenesis with a smaller reheating temperature. This is possible if there is a resonant enhancement of the generated asymmetry [13, 14, 15], some fine-tuning that violates the naturalness assumptions entering into the lower bound on the reheating temperature [16], or a violation of R-parity [17].

The second option is to accept a large reheating temperature and tackle the problems associated with the gravitino. One possibility is a very heavy gravitino that decays before BBN [18]. Alternatively, the gravitino could be very light, thus both avoiding a too large relic density and letting the NLSP decay before BBN [12]. The relic density of a gravitino LSP with mass in the GeV range can also be acceptable for rather large reheating temperatures [19], but in this case we have to protect BBN from the NLSP decays. This is possible if the NLSP decays relatively fast due to R-parity violation [20] or additional decay modes into hidden sector states [21, 22], if its decay products are only weakly interacting [23] or very low-energetic [24] and therefore harmless, or if its abundance is exceptionally small. The last alternative can occur for a stau NLSP [25, 26] in exceptional regions of parameter space, or for any NLSP whose abundance is diluted by entropy produced in late decays of another particle [27, 28, 29]. Of course, a combination of the various options is possible [30, 31, 32, 33].

In this work, we study a scenario where the gravitino is the LSP and forms the dark matter. The abundance of the NLSP is decreased by late-time entropy production. This dilution has the additional motivation that long-lived particles are anyway present in many extensions of the Standard Model (SM) and usually have to decay before BBN to avoid problems, producing entropy in the process. The solution of the gravitino problem may thus be viewed as a complimentary by-product of such an extension rather than

an additional complication of the Minimal Supersymmetric Standard Model (MSSM). Of the several candidates for the NLSP, we consider the lightest neutralino.

The paper is organised as follows. In Sec. 2, we review briefly the scenario of thermal leptogenesis with gravitino dark matter. Sec. 3 discusses the impact of entropy production on this scenario and the mechanism of entropy production by decaying matter. In Sec. 4, we present BBN constraints on a general neutralino NLSP after suitable dilution. We elaborate general properties of the scenario and strong constraints on the entropy-producing particle in Sec. 5, where we also consider as an example the saxion from the axion multiplet.

2 Leptogenesis and Gravitino Dark Matter...

In baryogenesis via standard thermal leptogenesis a cosmic lepton asymmetry is generated by CP-violating out-of-equilibrium decays of heavy right-handed Majorana neutrinos ν_{R}^i. Non-perturbative sphaleron processes [34, 35] convert the lepton asymmetry into a baryon asymmetry η_{B}. In the case of hierarchical masses the maximal resulting baryon-to-photon ratio of the Universe can be given as [36, 37]

$$\eta_{\mathrm{B}}^{\max} \simeq 9.6 \times 10^{-10}\, \Delta^{-1} \left(\frac{M_{\nu_{\mathrm{R}}^1}}{2 \times 10^9 \text{ GeV}}\right) \left(\frac{m_{\nu_{\mathrm{L}}^3}}{0.05 \text{ eV}}\right) \left(\frac{\kappa_0}{0.18}\right) \tag{1}$$

for the MSSM and weak washout, while the observed value lies in the range $5.89 \times 10^{-10} < \eta_{\mathrm{B}}^{\mathrm{obs}} < 6.49 \times 10^{-10}$ (2σ) [38]. $M_{\nu_{\mathrm{R}}^1}$ denotes the Majorana mass of the lightest right-handed neutrino. The given baryon asymmetry is maximal in the sense that the CP violation in the decays is chosen to be maximal [39]. Since thermal leptogenesis strongly favours hierarchical light neutrino masses [40], the mass $m_{\nu_{\mathrm{L}}^3}$ of the heaviest left-handed neutrino has to be close to $\sqrt{\Delta m_{31}^2} \simeq 0.050$ eV, using the best-fit value from neutrino data [41] and assuming a normal mass ordering. The efficiency factor κ_0 should be computed case-by-case by solving the relevant Boltzmann equations [42, 43, 44]. For zero initial ν_{R}^1 abundance in the small $M_{\nu_{\mathrm{R}}^1}$ regime [36], i.e. for $M_{\nu_{\mathrm{R}}^1} \lesssim 4 \times 10^{13}$ GeV, the maximal value is $\kappa_0^{\mathrm{peak}} \simeq 0.18$ [45]. This value is reached for

$$\widetilde{m}_{\nu_{\mathrm{L}}^1} \simeq m_* = \frac{8\pi^2 \sqrt{g_*}}{3\sqrt{10}} \frac{v^2}{M_{\mathrm{Pl}}} \simeq 1.6 \times 10^{-3} \text{ eV}\,, \tag{2}$$

where m_* is known as the equilibrium neutrino mass, $v \simeq 174$ GeV, and $M_{\mathrm{Pl}} \simeq 2.44 \times 10^{18}$ GeV. The effective neutrino mass

$$\widetilde{m}_{\nu_{\mathrm{L}}^1} = \frac{(m_{\mathrm{D}}^\dagger m_{\mathrm{D}})_{11}}{M_{\nu_{\mathrm{R}}^1}} \tag{3}$$

equals the mass of the lightest neutrino if the Dirac mass matrix $m_{\rm D}$ is diagonal. Its natural range is $m_{\nu_{\rm L}^1} < \widetilde{m}_{\nu_{\rm L}^1} < m_{\nu_{\rm L}^3}$. The parameter Δ denotes the dilution factor by entropy production after the decay of the right-handed neutrinos. It equals one in standard cosmology, while we will consider the general case $\Delta \geq 1$ later on.

There are some uncertainties entering (1). Possible spectator field uncertainties [46] and flavour effects [47, 48] are neglected, and the naive sphaleron conversion factor [49, 50] is used. We have assumed the particle content of the MSSM with $g_* = 228.75$ for the number of effectively massless degrees of freedom at high temperatures. To be conservative we consider the effects of the MSSM by a factor $2\sqrt{2}$ relative to the SM, which is valid for weak washout [37]. For strong washout this factor reduces to $\sqrt{2}$.

We see from (1) that leptogenesis in its minimal version as described above can generate the observed baryon-to-photon ratio of the Universe, because $\eta_{\rm B}^{\rm max}$ can exceed $\eta_{\rm B}^{\rm obs}$. On the other hand, it is clear that there is a lower bound $M_{\nu_{\rm R}^1} \gtrsim 2 \times 10^9$ GeV.

It is especially appealing for the considered neutrino mass range that leptogenesis can emerge as the unique source of the cosmological baryon asymmetry [40]. Wash-out processes may reduce a pre-existing asymmetry by two to three orders of magnitude for the situation of (1). Stronger washout decreases the efficiency factor and thus requires a larger right-handed neutrino mass to keep $\eta_{\rm B}^{\rm max} \geq \eta_{\rm B}^{\rm obs}$. For thermal leptogenesis, the bound on the lightest right-handed neutrino mass can be translated into a lower bound on the reheating temperature after inflation, $T_{\rm R} \gtrsim M_{\nu_{\rm R}^1}$. In the strong washout regime, i.e. for $\widetilde{m}_{\nu_{\rm L}^1} > m_*$, this changes to $T_{\rm R} \gtrsim 0.1\, M_{\nu_{\rm R}^1}$ [45], but we cannot relax the bound on the absolute value of $T_{\rm R}$, since in this case the efficiency factor decreases as well, requiring a larger $M_{\nu_{\rm R}^1}$.

The required high temperatures also lead to thermal production of a significant gravitino relic density

$$\Omega_{3/2}^{\rm tp} h^2 = m_{3/2}\, Y_{3/2}^{\rm tp}(T_0)\, \frac{s(T_0)\, h^2}{\rho_0}\,, \tag{4}$$

where $s(T_0)$ refers to today's entropy density of the Universe. Together with the Hubble constant h in units of 100 km Mpc^{-1} s^{-1} and today's critical density ρ_0, we obtain $s(T_0)h^2/\rho_0 \simeq 2.8 \times 10^8$ GeV^{-1}. The gravitino abundance for low temperatures $T_{\rm low} \ll T_{\rm R}$ is given by [51, 52]

$$Y_{3/2}^{\rm tp}(T_{\rm low}) \simeq \sum_{i=1}^{3} y_i g_i^2(T_{\rm R}) \left(1 + \frac{M_i^2(T_{\rm R})}{3m_{3/2}^2}\right) \ln\left(\frac{k_i}{g_i(T_{\rm R})}\right) \left(\frac{T_{\rm R}}{10^{10}\ {\rm GeV}}\right), \tag{5}$$

where the gauge couplings $g_i = (g', g, g_s)$, the gaugino mass parameters M_i as well as the constants $k_i = (1.266, 1.312, 1.271)$ and $y_i/10^{-12} = (0.653, 1.604, 4.276)$ are associated with the gauge groups U(1)$_{\rm Y}$, SU(2)$_{\rm L}$ and SU(3)$_{\rm C}$, respectively.

Without entropy production, the gravitino yield from thermal production at the present temperature $Y_{3/2}^{\mathrm{tp}}(T_0) = Y_{3/2}^{\mathrm{tp}}(T_{\mathrm{low}})$. With entropy production after the gravitino production in the early Universe,

$$Y_{3/2}^{\mathrm{tp}}(T_0) = \Delta^{-1}\, Y_{3/2}^{\mathrm{tp}}(T_{\mathrm{low}})\,. \tag{6}$$

As mentioned before, $\Delta = 1$ in standard cosmology. For reasons that we will explain below, we will consider late-time entropy production at $T \ll M_{\nu_{\mathrm{R}}^1}, T_{\mathrm{low}}$. Then the same dilution factor Δ appears in (1) and (6).[1]

From (4) and (5) we see that since the gravitino is the LSP, for fixed gaugino masses the relic gravitino density typically decreases for increasing gravitino mass. Assuming universal gaugino masses at the GUT scale, we can approximate

$$\Omega_{3/2}^{\mathrm{tp}} h^2 \simeq 0.11\,\Delta^{-1} \left(\frac{T_{\mathrm{R}}}{3\times 10^{8}\ \mathrm{GeV}}\right) \left(\frac{M_{\widetilde{g}}(m_{\mathrm{Z}})}{10^{3}\ \mathrm{GeV}}\right)^{2} \left(\frac{10\ \mathrm{GeV}}{m_{3/2}}\right). \tag{7}$$

Thus, for given reheating temperature and gaugino masses, we obtain a lower bound on the gravitino mass exploiting the requirement $\Omega_{3/2}h^2 \le \Omega_{\mathrm{DM}}h^2 = 0.112 \pm 0.007$ (2σ) [38].

In order to summarise the issues discussed so far, we combine (1) and (7) using the best-case relation $T_{\mathrm{R}} \simeq M_{\nu_{\mathrm{R}}^1}$ to eliminate the right-handed neutrino mass, arriving at

$$\eta_{\mathrm{B}}^{\max} \simeq 1.4\times 10^{-10} \left(\frac{\Omega_{3/2}^{\mathrm{tp}} h^2}{0.11}\right) \left(\frac{10^{3}\ \mathrm{GeV}}{M_{\widetilde{g}}(m_{\mathrm{Z}})}\right)^{2} \left(\frac{m_{3/2}}{10\ \mathrm{GeV}}\right) \times \left(\frac{m_{\nu_{\mathrm{L}}^3}}{0.05\ \mathrm{eV}}\right) \left(\frac{\kappa_0}{0.18}\right). \tag{8}$$

Note that the dilution factor Δ cancels out. Recalling the discussion after (1), $m_{\nu_{\mathrm{L}}^3}$ cannot be raised without lowering κ_0. Thus, even for the most optimistic scenario with $T_{\mathrm{R}} = 2\times 10^{9}$ GeV the gravitino mass is restricted to a rather large value $\gtrsim 40$ GeV. In other words, there is considerable tension between thermal leptogenesis and gravitino dark matter.

Even worse, the NLSP decay problem is a definite clash between both notions, since gravitino LSP masses larger than about 10 GeV are excluded in most cases. In the MSSM with conserved R-parity, the NLSP has to decay into the gravitino and SM particles. It decays typically with a long lifetime due to the extremely weak interactions of the gravitino. If these decays occur during or after BBN, the emitted SM particles can change the primordial abundances of the light elements [12, 53, 54]. Specific setups like the Constrained MSSM have been studied in [55, 56, 57, 58, 59]. Only in exceptional

[1] For $T_{\mathrm{R}} \gg M_{\nu_{\mathrm{R}}^1}$, it could be possible to produce entropy in between, so that only $\Omega_{3/2}$ would be diluted but not η_{B}.

regions of the parameter space [24, 25, 26], a stau NLSP not orders of magnitude heavier than the gravitino could be consistent with all constraints, allowing reheating temperatures $T_{\mathrm{R}} \sim 10^9$ GeV. Generically a conservative upper bound on the reheating temperature $T_{\mathrm{R}} \lesssim$ few 10^8 GeV is found. Since the maximally produced baryon asymmetry is too small, various NLSP candidates have been investigated more model-independently [60, 54] to identify best-case scenarios: a sneutrino [23, 61, 62, 63, 64, 65] actually could allow large enough reheating temperatures with reasonable masses due to its invisible decays. A stop [66, 67] or a general neutralino [68] are not reconcilable with thermal leptogenesis for masses below a TeV. They would definitely require $m_{3/2} < 10$ GeV.

Altogether, the strongest conflict between thermal leptogenesis and gravitino dark matter is found in the NLSP decay problem. This is embodied in (8) by the restriction to small gravitino masses, $m_{3/2} \leq 10$ GeV.

Entropy production after the freeze-out of the NLSP dilutes its density. Thus, late-time entropy production can naively resolve this conflict for any NLSP within or without a specific model. The relic density prior to its decay,

$$\Omega_{\mathrm{nlsp}} = \Delta^{-1}\, \Omega_{\mathrm{nlsp}}^{\mathrm{fo}}\,, \tag{9}$$

is reduced compared to its freeze-out density $\Omega_{\mathrm{nlsp}}^{\mathrm{fo}}$ by the dilution factor Δ, which is the same as in (1) and (6). In Sec. 4 we show how BBN constraints on a general neutralino with a gravitino LSP with $m_{3/2} = 100$ GeV are softened by $\Delta > 1$.

3 ... with late-time Entropy Production

3.1 Thermal Leptogenesis and Gravitino Yield with late-time Entropy Production

From (1) we see that a significant dilution $\Delta > 1$ can only be compensated by a larger $M_{\nu_{\mathrm{R}}^1}$, since all the other parameters are chosen already to be optimal. Due to the requirement $T_{\mathrm{R}} \gtrsim M_{\nu_{\mathrm{R}}^1}$ this gives a linear shift of the required reheating temperature. Since the gravitino density depends also linearly on the reheating temperature (5) and is diluted in the same way as the baryon density, such a compensation seems to give a trivial shift of the problem to higher reheating temperatures. However, there are aspects that do not show up in (1) and (8), in addition to the impact on the NLSP decay problem.

Most importantly, in the domain of large $M_{\nu_{\mathrm{R}}^1}$ washout processes reduce the efficiency factor κ_0 exponentially. In the case of hierarchical neutrinos, this domain corresponds to $M_{\nu_{\mathrm{R}}^1} > 4\times 10^{13}$ GeV. From this we would obtain $\Delta < 2 \times 10^4$. However, while at low $M_{\nu_{\mathrm{R}}^1}$ many numeric examples and an analytic approximation for κ_0 [45] exist in the literature, the situation for

larger $M_{\nu_R^1}$ is less well-studied. As an additional complication, for $T \gg 10^9$ GeV more spectator processes are in equilibrium and thus should be taken into account. Hence, it is not clear whether the maximal value κ_0^{peak} can be reached for $M_{\nu_R^1} \sim 4 \times 10^{13}$ GeV. Consequently, we expect an upper bound

$$\Delta < \Delta^{\max} \sim 10^3 \ldots 10^4 \,. \tag{10}$$

We would like to stress that this is an intrinsic bound of the problem. It is stronger than bounds from perturbativity of Yukawa couplings ($\Delta < 10^5$) or the requirement of a reheating temperature below the GUT scale ($\Delta < 10^7$).

We remark that according to Fig. 6b of [36] there is a much stronger bound with roughly $\Delta^{\max} < 10^2$ for quasi-degenerate neutrino masses. Thus, thermal leptogenesis with late-time entropy production requires hierarchical neutrinos even more than thermal leptogenesis already does.

Late-time entropy production leads to a strong reduction of the allowed parameter space for successful thermal leptogenesis. Since the required minimal $M_{\nu_R^1}$ is increased, the range of allowed values for κ_0 and the neutrino mass parameters is reduced. However, the same region of parameter space is already favoured by the need to keep the reheating temperature as low as possible in order to avoid the overproduction of gravitinos. Therefore, late-time entropy production does not reduce the parameter space of thermal leptogenesis with gravitino dark matter.

In (5) one has to consider the impact of the running couplings and masses due to the shift of the reheating temperature. For example, if we increase T_{R} from 3×10^9 GeV to 3×10^{13} GeV and choose $\Delta = 10^4$ to compensate, $\Omega_{3/2}^{\mathrm{tp}}$ decreases by 25%.[2] Note that this effect is unavoidable and softens the tension between thermal leptogenesis and gravitino dark matter already before considering the impact of entropy production on the NLSP decay problem.

Another possibility is a gravitino with such a small mass that it comes into thermal equilibrium after reheating. Then its relic abundance becomes independent of the reheating temperature, which allows $T_{\mathrm{R}} \gg M_{\nu_R^1}$. Taking into account the lower limit on the mass of a warm dark matter particle [69], it turns out that its relic energy density exceeds the observed dark matter density in standard cosmology. However, already a Δ of a few dilutes the gravitino sufficiently to make it viable warm dark matter again [31, 32]. For $\Delta \simeq 10^3$ it forms cold dark matter with $m_{3/2} \simeq 1$ MeV [33, 70, 71]. Note that for these small masses the NLSP decays before BBN, so that the decay problem is absent.

[2]Besides, the electroweak contributions double their contribution to the total yield to about 30%.

3.2 Entropy Production by decaying Matter

In this section we discuss briefly how decaying matter can produce considerable entropy in the early Universe [72, 73]. We consider a non-relativistic and long-lived particle species ϕ with chemical potential $\mu = 0$ in a flat Friedmann-Robertson-Walker Universe. When ϕ drops out of chemical equilibrium, its abundance $Y_\phi = n_\phi / s$ "freezes out", where n_ϕ denotes its number density and

$$s = \frac{2\pi^2}{45}\, g_*(T)\, T^3 \tag{11}$$

the entropy density of the Universe.[3] Y_ϕ could also be generated from inflaton decay or thermally after reheating, if ϕ never enters chemical equilibrium. The contribution of non-relativistic particles to the energy density decreases as $\rho_{\text{mat}} \propto R^{-3}$, where R denotes the scale factor. Since the energy density of radiation in the Universe,

$$\rho_{\text{rad}} = \frac{\pi^2}{30}\, g_*(T)\, T^4\,, \tag{12}$$

decreases $\propto R^{-4}$, $\rho_{\text{mat}}/\rho_{\text{rad}}$ grows $\propto R$. Since R grows with time, at some time $t_\phi^=$ or temperature $T_\phi^=$ the unstable species ϕ comes to dominate the energy density automatically, if its lifetime $\tau_\phi > t_\phi^=$. If the Universe has been dominated by radiation before, it enters a phase of matter domination that lasts roughly till ϕ exponentially decays at τ_ϕ. Here we assume that everything released by the particle decay is rapidly thermalized, i.e. on timescales $\Delta t \ll H^{-1} \simeq$ expansion time. At some intermediate time $t \simeq t_\phi^= \,(\tau_\phi/t_\phi^=)^{3/5}$ the radiation produced in decays of ϕ starts to become the dominant component of the radiation energy density. The temperature of the Universe begins to fall more slowly, $T \propto R^{-3/8}$, than the usual $T \propto R^{-1}$. The Universe is never reheated, since the temperature decreases at all times. From $t \simeq t_\phi^= \,(\tau_\phi/t_\phi^=)^{3/5}$ till $t \simeq \tau_\phi$, the entropy per comoving volume S is growing $\propto R^{15/8}$. At τ_ϕ the Universe becomes purely radiation-dominated again with $T \propto R^{-1}$ and a temperature $T_\phi^{\text{dec}} = T(\tau_\phi)|_{\text{rad-dom}}$, where we use the time–temperature relation for a radiation-dominated Universe,

$$t|_{\text{rad-dom}} = \left(\frac{45}{2\pi^2 g_*(T)}\right)^{\frac{1}{2}} M_{\text{Pl}} T^{-2}\,. \tag{13}$$

This is the temperature after significant entropy production T^{after}, which would be identified as the reheating temperature in the approximation of simultaneous decay of all ϕ particles. We identify $T^{\text{after}} = T_\phi^{\text{dec}}$. If ρ_ϕ never dominates over ρ_{rad}, ϕ decays never produce a significant amount of entropy relative to the initial entropy. Then the produced entropy is negligible.

[3] For simplicity we use g_* only, since the temperatures occurring in this work are above 1 MeV, where $g_{*S} = g_*$.

However, if ρ_ϕ dominates over $\rho_{\rm rad}$ before τ_ϕ, the produced entropy dilutes significantly any relic density by a factor Δ.

The dilution factor Δ is defined as the ratio of entropy per comoving volume after ϕ decay S_f over the initial entropy per comoving volume S_i and can be expressed as

$$\Delta = \frac{S_f}{S_i} \simeq 0.82 \left\langle g_*^{1/3} \right\rangle^{3/4} \frac{m_\phi Y_\phi \tau_\phi^{1/2}}{M_{\rm Pl}^{1/2}} \,, \tag{14}$$

where the angle brackets indicate the appropriately-averaged value of $g_*^{1/3}$ over the decay interval. We see how Δ is determined by the properties of the unstable particle, i.e. its mass m_ϕ and lifetime τ_ϕ. Meanwhile it is assumed that $\tau_\phi > t_\phi^=$. The pre-decay abundance Y_ϕ of the unstable particle depends on both its interactions and the earlier cosmology.

For convenience we would like to rephrase (14) in terms of temperatures. Without entropy production after the generation of the pre-decay abundance, it is constant till the particle decays. With $\rho = mn$ we find

$$m_\phi Y_\phi = \frac{\rho_\phi}{s} = \frac{\rho_{\rm rad}}{s}\bigg|_{T=T_\phi^=} = \frac{3}{4} T_\phi^= \,, \tag{15}$$

where we have used (11), (12) and $\rho_\phi = \rho_{\rm rad}$ at $T_\phi^=$.

Using (13) we can replace the particle lifetime in (14) as

$$\tau_\phi^{\frac{1}{2}} = \left(\frac{45}{2\pi^2}\right)^{\frac{1}{4}} g_*^{-\frac{1}{4}}(T_\phi^{\rm dec})\, M_{\rm Pl}^{\frac{1}{2}} \left(T_\phi^{\rm dec}\right)^{-1} . \tag{16}$$

Plugging (15) and (16) into (14) we obtain

$$\Delta = 0.75 \frac{\left\langle g_*^{1/3} \right\rangle^{3/4}}{g_*^{1/4}(T_\phi^{\rm dec})} \frac{T_\phi^=}{T_\phi^{\rm dec}} \,. \tag{17}$$

This linear growth in temperature can also be expressed in terms of energy densities, since

$$\rho_\phi = n_\phi m_\phi = s Y_\phi m_\phi = \frac{2\pi^2}{45} \frac{3}{4} g_*(T)\, T^3\, T_\phi^= \,, \tag{18}$$

where we have used (11) and (15). Taking this together with (12) we find

$$\frac{\rho_\phi}{\rho_{\rm rad}} = \frac{T_\phi^=}{T} \,. \tag{19}$$

Thus, for $T = T_\phi^{\rm dec}$ we see that

$$\frac{T_\phi^=}{T_\phi^{\rm dec}} = \frac{\rho_\phi}{\rho_{\rm rad}}(T_\phi^{\rm dec}) \,, \tag{20}$$

where $\rho_{\rm rad}$ is the density of the "old" radiation, i.e. it does not include the radiation from ϕ decays.

The standard Big Bang model has been tested thoroughly up to temperatures around 1 MeV, where BBN occurs. Investigations of the thermalization of neutrinos produced in ϕ decays or subsequent thermalization processes lead to lower limits on the temperature of the Universe after the entropy production $T^{\rm after}$ [74]. Neutrinos, which can thermalize through weak interactions only, are most important. All other SM particles thermalize much faster due to their stronger interactions. The bounds found are in the range

$$T_\phi^{\rm dec} > T_{\rm min}^{\rm after} \simeq (0.7 \ldots 4)\ {\rm MeV} \tag{21}$$

where weaker bounds come from BBN calculations [75, 76] and stronger bounds rely on the neutrino energy density [77, 78] exploiting overall best fits for cosmological parameters. We take $T_\phi^{\rm dec} \geq 4$ MeV $\sim T_{\rm BBN}$ as lower bound.

Going back in time, thus towards higher temperatures, the first cosmological event important for our scenario is the freeze-out of the NLSP.[4] Standard computations of relic abundances rely on the assumption of radiation domination during freeze-out. If the Universe is dominated by matter during NLSP freeze-out, the NLSP relic abundance is increased. Taking the later dilution by entropy production into account, the overall effect remains a reduction [79]. The effects of different cosmological scenarios on relic densities have been studied [80, 81, 82] and there are computer codes [83]. In particular, the neutralino has been investigated, also considering the production of neutralinos in the decay of a dominating matter particle [84, 85, 86, 87]. Since it is the easiest case to study, we take $T_\phi^{=} < T_{\rm nlsp}^{\rm fo}$. Thereby the Universe is radiation-dominated during NLSP freeze-out happening at $T_{\rm nlsp}^{\rm fo}$ and the standard computations hold.[5] Later we will find that the window between BBN and NLSP freeze-out is favoured intrinsically by the scenario.

Sticking to this particular window, we can evaluate (17),

$$\Delta \simeq 0.75 \times 10^3 \left(\frac{m_{\rm nlsp}}{100\ {\rm GeV}}\right)\left(\frac{4\ {\rm MeV}}{T_\phi^{\rm dec}}\right), \tag{22}$$

where we have plugged in $T_\phi^{=} = T_{\rm nlsp}^{\rm fo} \simeq m_{\rm nlsp}/25$ and $\left\langle g_*^{1/3}\right\rangle \simeq 2.2 \simeq g_*^{1/3}(T_\phi^{\rm dec})$ with $g_*(T_\phi^{\rm dec}) = 10.75$, exploiting the fact that for 4 MeV $\leq T \leq$ 4 GeV the effective relativistic degrees of freedom are known [88]. If we

[4] The QCD phase transition occuring between BBN and NLSP freeze-out seems not to deliver any constraint on our scenario.

[5] Using the simple estimate $H(T_{\rm nlsp}^{\rm fo}) \sim \Gamma(T_{\rm nlsp}^{\rm fo})$, where Γ is the rate of NLSP annihilations, one finds that for $T_\phi^{=} = T_{\rm nlsp}^{\rm fo}$ the NLSP abundance is increased by a factor of only $\sqrt{2}$ compared to the standard case of radiation domination, while the freeze-out temperature stays nearly constant.

compare (22) and (10), we see that the cosmological window between BBN and NLSP freeze-out is not only the first and easiest but also sufficiently large to produce enough entropy to come close to the upper limit on Δ set by thermal leptogenesis itself.

This discussion assumes that there is no further entropy production after the generation of Y_ϕ. Otherwise, Y_ϕ would be diluted like any other relic abundance, i.e. $Y_\phi \to Y'_\phi = \Delta_1^{-1} Y_\phi$. There are two possibilities for the impact of such an earlier entropy increase $\Delta_1 > 1$. i) Despite the dilution, ϕ dominates the Universe for some time. Then the later entropy production by the decay of ϕ is simply reduced by a factor Δ_1, as we see from (14) since lifetime and mass of the unstable particle are unchanged. ii) The relic abundance of ϕ becomes so small that the particle never dominates the energy density of the Universe. Then (14) does not hold and $S \simeq$ const., i.e. $\Delta = 1$.

After an arbitrary number of late events of entropy production Δ_i labeled by $i = 1, 2, \ldots$, where the index implies a time-ordering with larger i corresponding to later decays, the total dilution factor is

$$\Delta_{\mathrm{tot}} = \prod_i \Delta_i \,. \tag{23}$$

Here, "late" indicates that all decays happen after the freeze-out of all unstable particles supposed to produce significant entropy, so that their relic abundances are diluted by each earlier decay. This implies

$$\Delta_i = \max\left\{ \frac{\Delta_i(\Delta_{j<i} = 1)}{\prod_{j<i} \Delta_j}\,, 1 \right\}, \tag{24}$$

where $\Delta_i(\Delta_{j<i} = 1)$ refers to the dilution factor obtained from (14) without considering the other dilutions in the calculation of Y_ϕ. As mentioned, we set $\Delta_i = 1$, if a decaying particle does not come to dominate the energy density of the Universe. One can easily convince oneself that the total dilution is simply given by the largest individual dilution factor,

$$\Delta_{\mathrm{tot}} = \max\left\{\Delta_i(\Delta_{j<i} = 1)\right\}. \tag{25}$$

The upper bound (10) limits Δ_{tot}. The dilution of the NLSP abundance can be smaller than Δ_{tot} if some decays happen before NLSP freeze-out. Thus, we see from (24) with (10) how our requirement of sufficient entropy production after NLSP freeze-out restricts the possibility of earlier entropy production.

4 BBN Constraints on a Diluted Neutralino NLSP

In this section we present constraints from Big Bang Nucleosynthesis on a neutralino NLSP in the case of a gravitino with a mass of $m_{3/2} = 100$ GeV

being the LSP. We investigated those bounds excluding the possibility of entropy production in [68] and found for masses below a TeV a maximal gravitino mass of a few GeV. In the following we assume that the neutralino is diluted after its freeze-out by a factor $\Delta = 10^3$. It is trivial to infer the impact of arbitrary Δs. BBN constraints on a stau NLSP with Δ up to 2×10^4 have been studied in [27, 28], where it has been found that interesting parameter regions are allowed for dilution factors $\Delta \sim 10^3$.

For a discussion of the neutralino decay channels, branching ratios and more details we refer to [68]. To determine model-independent constraints within the MSSM we take all points that are not ruled out by LEP up to a mass of 2 TeV, while we fix the masses of the sfermions to be above 2 TeV. To keep our analysis as general as possible we do not fix all supersymmetric parameters according to a specific scenario, but instead we set the soft SUSY breaking parameters at the low energy scale. We keep the majority of the parameters fixed and vary the gaugino and Higgsino mass parameters to study how the lifetime and number density vary with the mass and composition of the lightest neutralino. We plot these points against the hadronic and electromagnetic BBN bounds in Figs. 1–3. The bounds are taken from [53] and the different curves are explained in the figure caption. The vertical axis corresponds to the fraction of the energy density that decays to electromagnetic or hadronic products. A $\Delta > 1$ shifts all points downwards on this axis by a factor of Δ. Therefore it is easy to infer constraints for arbitrary Δ once a plot with fixed Δ is given.

Hadronic bounds are generally more constraining. However, it has been found that large gravitino masses, for which light neutralinos have a low hadronic branching ratio, are excluded by the electromagnetic bounds.

In Fig. 1 we consider a mixed bino-wino NLSP. The large dip corresponds to resonant annihilation into the pseudo-scalar Higgs, which happens for our choice of parameters at a neutralino mass $m_\chi \sim 1150$ GeV. To increase $\eta_{\mathrm{B}}^{\max}$ we are more interested in the region of small NLSP masses, since small NLSP masses allow more easily for small gluino masses in (8).

Thanks to the dilution by entropy production the wino overcomes the electromagnetic bounds for any mass even for masses close to the gravitino mass. If the neutralino is mainly bino the electromagnetic bounds are more involved. For a bino-like neutralino, masses below about 450 GeV are excluded. Smaller and smaller masses become allowed when the wino component increases, so that there is allowed space for binos with a non-negligible wino component and $m_\chi \sim 200$ GeV or even smaller masses.

The hadronic bounds exclude most of the parameter space for a bino-wino with dominant bino component even with $\Delta = 10^3$. The mixed bino-wino states with $m_\chi \sim 200$ GeV mentioned above are found on the less conservative ${}^6\mathrm{Li}/{}^7\mathrm{Li}$ exclusion line for a decaying particle of 100 GeV mass. Thus we find many points that should not be considered as strictly excluded with masses around 200 GeV and also mixed bino-wino states that

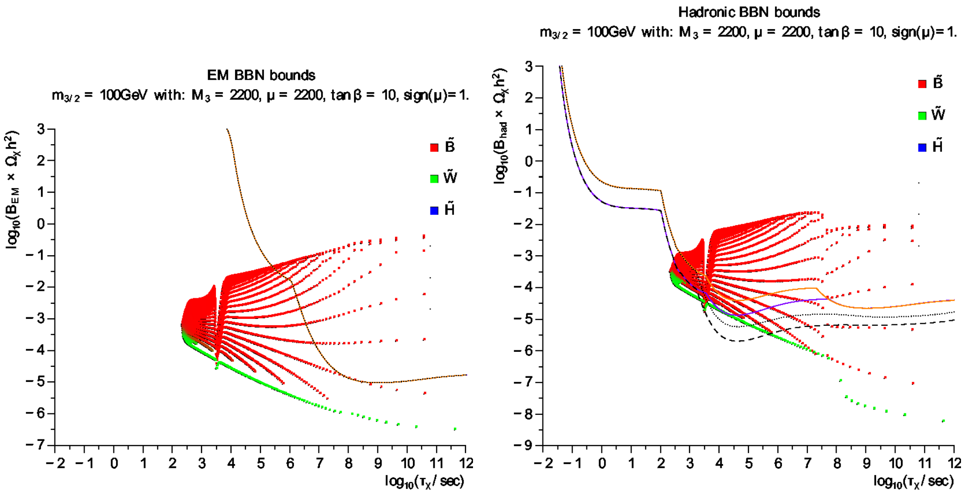

Figure 1: Energy density of the bino-wino neutralino decaying into electromagnetic/hadronic products compared with the BBN electromagnetic (left) and hadronic constraints (right) for the case of a 100 GeV gravitino mass and a dilution factor $\Delta = 10^3$. The bounds are taken from [53]: the continuous (dashed) lines correspond to more (less) conservative bounds for the ^{6}Li to ^{7}Li ratio, and the region between the curves should not be considered as strictly excluded. The red/upper and violet/lower curves in the hadronic plots are the constraints for 1 TeV and 100 GeV decaying particle mass, respectively. The mass increases from right to left as heavier particles decay faster. The composition goes from bino at the top to wino at the bottom while the colours give the dominant component. The deformation between the left and right panel is due to the mass dependence of the hadronic branching ratio with lighter NLSPs having lower branching ratios to hadrons. In contrast the electromagnetic branching ratio is always nearly one.

are allowed with masses smaller than 200 GeV. Winos with $m_\chi \lesssim 400$ GeV overcome even any less conservative bound. For 400 GeV $\lesssim m_\chi \lesssim$ 1100 GeV the wino could violate the less conservative bound, while even larger masses become allowed again.

Altogether, the situation is qualitatively different for bino and wino. While the wino safely overcomes all bounds, especially at low masses, a bino-like neutralino with reasonable mass stays excluded even for much larger dilution factors that would be in contradiction with successful thermal leptogenesis (10). However, there is also some space for bino-wino mixed states that are mainly bino with masses below 200 GeV.

In Fig. 2 we consider a mixed bino-Higgsino NLSP. The dip is broader in this case, since the Higgsino component that couples to the pseudo-scalar Higgs is larger. Thanks to the dilution the Higgsino overcomes the electromagnetic bounds like the wino for all masses, even though Δ should not be much smaller than roughly 10^2 to allow for light Higgsino neutralinos. For the bino the situation is comparable to the case of mixed bino-wino. No

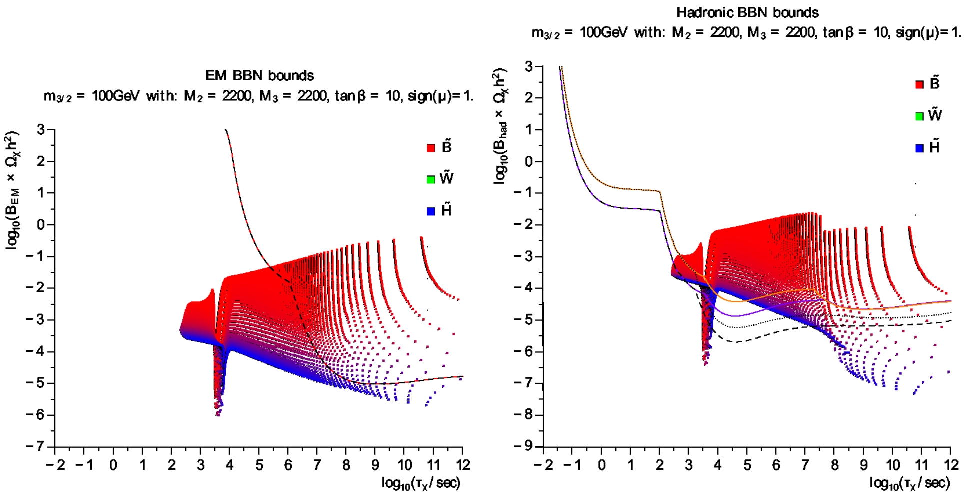

Figure 2: Energy density of the bino-Higgsino neutralino decaying into electromagnetic/hadronic products compared with the BBN electromagnetic (left) and hadronic constraints (right) for the case of a 100 GeV gravitino mass and a dilution factor $\Delta = 10^3$. The bounds are taken from [53]: the continuous (dashed) lines correspond to more (less) conservative bounds for the ^{6}Li to ^{7}Li ratio, and the region between the curves should not be considered as strictly excluded. The red/upper and violet/lower curves in the hadronic plots are the constraints for 1 TeV and 100 GeV decaying particle mass, respectively. The mass increases from right to left as heavier particles decay faster. The composition goes from bino at the top to Higgsino at the bottom while the colours give the dominant component. The deformation between the left and right panel is due to the mass dependence of the hadronic branching ratio with lighter NLSPs having lower branching ratios to hadrons. In contrast the electromagnetic branching ratio is always nearly one.

mixed bino-Higgsino state with a dominant bino component is allowed with masses as low as 200 GeV, though.

Again, the hadronic bounds exclude most of the bino parameter space. Exceptions are found in the dip and at very large masses. There are states with comparable bino and Higgsino components and $m_\chi \gtrsim 200$ GeV—thus not excluded by the electromagnetic bounds—violating the less conservative hadronic bound. Higgsino neutralinos lighter than 250 GeV escape even these constraints, while they are excluded for 670 GeV $\lesssim m_\chi \lesssim$ 1100 GeV.

Altogether, we find that for mixed bino-Higgsino only states that are mainly Higgsino allow for preferrable small masses but then even down to the gravitino mass. Considering only the conservative hadronic bound from the ^{6}Li to ^{7}Li ratio, in addition maximally mixed states with masses in the region around 230 GeV become allowed. $\Delta > 10^3$ would allow for larger bino components in the mixed bino-Higgsino.

In Fig. 3 we consider a mixed wino-Higgsino NLSP. Thanks to the dilution it overcomes the electromagnetic bounds for all masses and mixings.

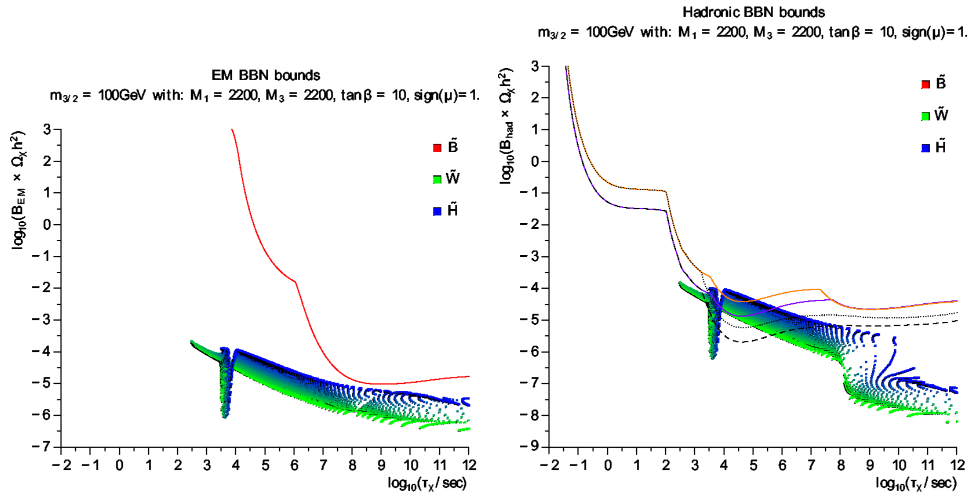

Figure 3: Energy density of the wino-Higgsino neutralino decaying into electromagnetic/hadronic products compared with the BBN electromagnetic (left) and hadronic constraints (right) for the case of a 100 GeV gravitino mass and a dilution factor $\Delta = 10^3$. The bounds are taken from [53]: the continuous (dashed) lines correspond to more (less) conservative bounds for the ^{6}Li to ^{7}Li ratio, and the region between the curves should not be considered as strictly excluded. The red/upper and violet/lower curves in the hadronic plots are the constraints for 1 TeV and 100 GeV decaying particle mass, respectively. The mass increases from right to left as heavier particles decay faster. The composition goes from Higgsino at the top to wino at the bottom while the colours give the dominant component. The deformation between the left and right panel is due to the mass dependence of the hadronic branching ratio with lighter NLSPs having lower branching ratios to hadrons. In contrast the electromagnetic branching ratio is always nearly one.

We are especially interested in the small mass region. By vertical shifts of all points we can search for the minimal dilution factor $\Delta^{\min}$ to overcome the electromagnetic bounds at small masses. A wino close to the gravitino mass becomes allowed for

$$\Delta^{\min}_{\widetilde{W}} \simeq 25\,. \tag{26}$$

Larger Δ allows for more wino masses and eventually for light Higgsinos at

$$\Delta^{\min}_{\widetilde{H}} \simeq 90\,. \tag{27}$$

Considering the hadronic constraints, winos with $m_\chi < 400$ GeV and Higgsinos with $m_\chi < 200$ GeV satisfy all bounds. Wino-Higgsinos with larger mass can be in conflict with the less conservative bound, and the wino overcomes the more conservative one completely. Disregarding the dip, Higgsinos are excluded for a window 700 GeV $\lesssim m_\chi \lesssim$ 1300 GeV.

In summary, entropy production after NLSP freeze-out can allow for a gravitino LSP of 100 GeV mass with a light neutralino NLSP. This reconciles thermal leptogenesis and gravitino dark matter within the scenario of

a light neutralino NLSP. However, this depends on the composition of the lightest neutralino. The wino is the best case due to its small freeze-out abundance. Also a light Higgsino becomes allowed for reasonable dilution factors. A light bino-like neutralino, which is typical for the Constrained MSSM, stays excluded for $m_{3/2} = 100$ GeV even if the possibility of entropy production after freeze-out is exploited.

Coming back to thermal leptogenesis, strictly speaking none of the points in the figures allows for a sufficiently high reheating temperature, since the gluino mass has been fixed at 2.2 TeV. However, in the parameter space regions with smaller neutralino masses, the gluino mass can be lowered without affecting our considerations at all. Consequently, all allowed points with a neutralino mass below a TeV can be compatible with thermal leptogenesis.

5 Search for a viable Candidate

5.1 General Requirements

In this section we discuss candidates for the entropy-producing particle ϕ of the previous sections. After enumerating the required properties in general, we exemplify in detail an implementation of the scenario with the axion multiplet.

To dilute the NLSP relic density, ϕ must *i) decay after NLSP freeze-out.* But, for sure, it *ii) decays before BBN.* Thus the lifetime τ_ϕ or equivalently the decay temperature T_ϕ^{dec} is constrained to a window. The particle has to be *iii) produced in the early Universe such that it dominates the energy density before BBN.* Meanwhile we stick to the case where its relic density *iv) does not come to dominate before NLSP freeze-out.* Thus the relic density prior to its decay $\rho_\phi = Y_\phi m_\phi s$ is also constrained to a window. Requirements iii)+iv) imply that the dominance of ϕ has to grow with the expansion, which is true for non-relativistic matter. So ϕ is implicitly assumed to become non-relativistic before BBN. The requirements i)+ii) and iii)+iv) constrain two different quantities τ_ϕ and ρ_ϕ, which are determined in different ways but by the same properties of ϕ, namely its couplings and mass.

Requirements i)+ii) constrain only the total decay rate $\Gamma_\phi^{\mathrm{tot}} = \tau_\phi^{-1}$. In fact, the branching ratios of ϕ into the LSP and NLSP are also constrained. The branching ratio into the NLSP $B_{\phi\to\mathrm{nlsp}+\ldots}$ must be so small that the *v) NLSP decay problem is not reintroduced by the decay.* Branching ratios into the LSP are always restricted by overproduction. Especially when the LSP is already produced thermally as in our scenario of thermal leptogenesis with gravitino dark matter, ϕ should *vi) not produce too many LSPs in its decays.* Since ϕ even dominates the energy density of the Universe at its decay, the requirements v)+vi) force the corresponding branching ratios to be—at least—close to zero.

In addition, ϕ must be *vii) compatible with gravitino dark matter.* For

No.	Requirement	Comment
i	$T_\phi^{\rm dec} < T_{\rm nlsp}^{\rm fo}$	to have effect on $\Omega_{\rm nlsp}$
ii	$T_\phi^{\rm dec} > T_{\rm BBN}$	not to spoil BBN
iii	$\frac{\rho_\phi}{\rho_{\rm rad}}(T_\phi^{\rm dec}) > 1$	$\mathcal{O}(10) < \Delta < 10^4$
iv	$\frac{\rho_\phi}{\rho_{\rm rad}}(T_{\rm nlsp}^{\rm fo}) < 1$	for standard NLSP freeze-out
v	$B_{\phi\to{\rm nlsp}+\dots} \simeq 0$	from NLSP decay problem
vi	$B_{\phi\to\Psi_{3/2}+\dots} \simeq 0$	from overproduction ($\Omega_{3/2}^{\rm tp} \simeq \Omega_{\rm DM}$)
vii	e.g. $\tau_{3/2} \gg t_0$	compatibility with gravitino dark matter
viii	ii) and v)–vii)	for by-products; no new problems

Table 1: List of requirements for our scenario of entropy produced by ϕ to dilute the NLSP.

example, the gravitino would become unstable due to the existence of ϕ, if it could decay into ϕ. This would take away the explanation for the observed dark matter abundance, if the gravitino lifetime were too short, or by itself be in conflict with other observations.

Finally, *viii) unavoidable by-products of ϕ have to be harmless.* For example, such by-products are the supermultiplet partners in SUSY. They are harmless, if they do not violate ii) or vii), are free of the problems solved by v)+vi) and do not introduce new problems on their own.

Altogether, the properties of ϕ seem to be highly constrained. We summarise the requirements in Tab. 1. The number of free parameters—mass and couplings—is finite. Since they enter in different ways for different constrained quantities, it is not a matter of course that the scenario of late-time entropy production is viable at all. Especially if Y_ϕ is produced thermally via scatterings, the same coupling might be responsible for the production and the late decay.

On the other hand, many extensions of the SM contain or predict superweakly interacting and hence long-lived particles. Such particles generically satisfy i), if not by definition. In order to ensure that they are harmless, one usually demands that they decay before BBN conform to ii). Thermal leptogenesis places the upper limit (10) on the maximally allowed dilution, which implies that for decay right before BBN iv) has to hold at least approximately. Considering high reheating temperatures and the growth of $\rho_{\rm mat}/\rho_{\rm rad} \propto R$, it is probable that the energy density of late-decaying particles dominates over the radiation energy density at their decay. Thus, iii) can be considered as fulfilled generically, which in fact normally poses a problem. Besides, the decay into superparticles usually has to be suppressed

in order to avoid producing too much dark matter and further late-decaying particles like the NLSP or the gravitino, in case it is not the LSP. Consequently, v) and vi) are generic, too, possibly amended by $B_{\phi\to\mathrm{lsp}+\ldots} \simeq 0$, if the gravitino is not the LSP. In any case the scenario has to be compatible with whatever is supposed to form the dark matter, so that vii) is generic. Also viii) arises as a generic requirement on any late-decaying particle and is particularly constraining in supersymmetric models.

In summary, ϕ is severely constrained such that the scenario of entropy production to dilute the NLSP density might appear unappealing. However, in extensions of the SM containing long-lived particles, in principle i)+ii) and v)–viii) are no new requirements and are present—in appropriate form—without considering entropy production at all. If $T_\phi^{\mathrm{dec}} \sim T_{\mathrm{BBN}}$, successful thermal leptogenesis favours the situation of iv). Finally, for the corresponding high reheating temperatures iii) is generic. Thus, all the requirements of Tab. 1 either have to be fulfilled or are generically fulfilled. In other words, the solution of the generic problems of long-lived particles may well cause the entropy production desired to solve the NLSP decay problem and thereby reconcile thermal leptogenesis and gravitino dark matter.

With a specific candidate at hand the details have to be worked out. One has to determine whether a candidate is excluded, not useful or can be the solution and how generically this is true. As an example, we investigate the axion multiplet, which is motivated by a completely disconnected problem of the SM.

5.2 Example: Axion Multiplet

The strong CP problem of the SM can be solved by the Peccei-Quinn (PQ) mechanism [89, 90]. An additional global $\mathrm{U}(1)_{\mathrm{PQ}}$ symmetry referred to as PQ symmetry, which is broken spontaneously at some PQ scale, can explain the smallness of the CP-violating Θ-term in QCD. The pseudo Nambu-Goldstone boson associated with this spontaneous symmetry breaking is called axion [91, 92]. It has not yet been observed. However, axion physics provides a lower limit [88, 93] on the axion decay constant,

$$f_a \gtrsim 6 \times 10^8 \ \mathrm{GeV}\,. \tag{28}$$

We identify the PQ scale with the axion decay constant.[6] Since our considered reheating temperatures are relatively large, it is probable that $T_{\mathrm{R}} > T_{\mathrm{PQ}} \sim f_a$.

If the PQ mechanism is supersymmetrised [94, 95], the axion a is part of a supermultiplet, the axion multiplet. It consists of the axino $\widetilde{a}$ containing

[6] This is equivalent to the choice $N = 1$, where N characterizes the colour anomaly of $\mathrm{U}(1)_{\mathrm{PQ}}$. With this choice we also avoid possible problems with topological defects. In the working scenario at the end the axion abundance from strings is negligible. For $N \neq 1$ our formulae hold, but $f_a = f_{\mathrm{PQ}}/N \neq f_{\mathrm{PQ}}$.

the fermionic degrees of freedom, the axion itself and the saxino or saxion $\phi_{\rm sax}$, which is an additional real scalar degree of freedom.

We investigate $\phi_{\rm sax}$ as candidate for the entropy-producing particle ϕ of the previous sections. Thus the axion and the axino are unavoidable by-products, while it will become clear why they are no candidates themselves. However, the saxion is motivated from the strong CP problem and not from our scenario.

5.2.1 Thermally Produced Multiplet

After reheating, reactions like $q\bar{q} \leftrightarrow g\phi_{\rm sax}$ and $gg \leftrightarrow g\phi_{\rm sax}$ drive the saxion into thermal equilibrium if the reheating temperature is larger than its decoupling temperature [96]

$$T_{\rm sax}^{\rm dcp} \simeq 10^{11}\ {\rm GeV} \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^2 \left(\frac{0.1}{\alpha_{\rm s}}\right)^3 , \tag{29}$$

where $\alpha_{\rm s} = g_s^2(\mu)/4\pi$ evaluated at the scale relevant for the processes under consideration. The equilibrium saxion abundance is given by

$$Y_{\rm sax}^{\rm eq} = \frac{45\,\zeta(3)}{2\pi^4 g_*(T_{\rm sax}^{\rm dcp})} \simeq 1.21 \times 10^{-3} . \tag{30}$$

Throughout this paper we assume for simplicity the particle content of the MSSM when we determine for example the relativistic degrees of freedom in the Universe. The numerical changes from adding the axion multiplet, for instance, are tiny as we can see in all estimates.

The saxion becomes non-relativistic at a temperature $T_{\rm sax}^{\rm nr} \simeq 0.37\, m_{\rm sax}$ around its mass [96]. From $0.37\, m_{\rm sax} \simeq T_{\rm sax}^{\rm nr} > T_{\rm sax}^{=} = T_{\rm nlsp}^{\rm fo} \simeq m_{\rm nlsp}/25$ would arise a lower bound

$$m_{\rm sax} > \frac{m_{\rm nlsp}}{9.25} . \tag{31}$$

We will find that this is weaker than the lower bound on the saxion mass from early enough decay (36) and thus in nearly all cases and at least in the interesting ones does not yield any constraint. With (15) we see that if the saxion lives long enough, it dominates the energy density of the Universe below the temperature

$$T_{\rm sax}^{=} = \frac{4}{3} Y_{\rm sax}^{\rm eq} m_{\rm sax} \simeq 1.6\ {\rm GeV} \left(\frac{m_{\rm sax}}{1\ {\rm TeV}}\right) . \tag{32}$$

We avoid matter domination during NLSP freeze-out by requiring $T_{\rm sax}^{=} < T_{\rm nlsp}^{\rm fo} \simeq m_{\rm nlsp}/25$, which gives an upper bound

$$m_{\rm sax} < 2.5\ {\rm TeV} \left(\frac{m_{\rm nlsp}}{10^2\ {\rm GeV}}\right) . \tag{33}$$

As we know (17), considerable entropy is produced only, if $T_{\rm sax}^{\rm dec} \ll T_{\rm sax}^{=}$. The saxion decay temperature can be derived from (13) with $T = T_{\rm sax}^{\rm dec}$ and $t = 1/\Gamma_{\rm sax}^{gg}$, where

$$\Gamma_{\rm sax}^{gg} \simeq \frac{\alpha_{\rm s}^2 m_{\rm sax}^3}{128\pi^3 f_a^2} \tag{34}$$

is the width of the dominant saxion decay into two gluons [96]. This yields [97]

$$T_{\rm sax}^{\rm dec} \simeq 53\ {\rm MeV} \left(\frac{10^{12}\ {\rm GeV}}{f_a}\right) \left(\frac{m_{\rm sax}}{1\ {\rm TeV}}\right)^{\frac{3}{2}} \left(\frac{\alpha_{\rm s}}{0.1}\right) \left(\frac{10.75}{g_*(T_{\rm sax}^{\rm dec})}\right)^{\frac{1}{4}} . \tag{35}$$

Here, $\alpha_{\rm s}$ has to be evaluated at $m_{\rm sax}$. As we do not consider an extremely large range of saxion masses, $\alpha_{\rm s}(m_{\rm sax})$ does not vary significantly. Besides, in the range of parameters considered, $g_*(T_{\rm sax}^{\rm dec})$ remains approximately constant. Therefore, we drop the explicit dependence on $\alpha_{\rm s}$ and $g_*(T_{\rm sax}^{\rm dec})$ in the following equations. Together with the bound (21) from early enough decay, we obtain the lower limit

$$m_{\rm sax} > 180\ {\rm GeV} \left(\frac{T_{\rm min}^{\rm after}}{4\ {\rm MeV}}\right)^{\frac{2}{3}} \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{\frac{2}{3}} . \tag{36}$$

If we compare this lower bound with (31), we see that (36) is stronger as long as

$$\left(\frac{f_a}{1.5\times 10^{10}\ {\rm GeV}}\right)^{\frac{2}{3}} \gtrsim \left(\frac{m_{\rm nlsp}}{10^2\ {\rm GeV}}\right) . \tag{37}$$

In any case, the saxion mass is constrained to a window. Since there is no additional source of SUSY breaking, one expects $m_{\rm sax} \sim m_{\rm susy}$ and thus a saxion mass in the TeV range, i.e. $10^2\ {\rm GeV} \lesssim m_{\rm sax} \lesssim 1\ {\rm TeV}$. Thus, from this discussion one might conclude that the requirements i)–iv) of Tab. 1 are naturally fulfilled.

Plugging (32) and (35) into (17) we obtain

$$\Delta \simeq 13 \left\langle g_*^{1/3} \right\rangle^{3/4} \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right) \left(\frac{1\ {\rm TeV}}{m_{\rm sax}}\right)^{\frac{1}{2}} . \tag{38}$$

For simplicity we replace $\left\langle g_*^{1/3} \right\rangle$ for the moment by 2.2, the value estimated for (22). We plug in the bounds on the saxion mass (33) and (36) to find

$$14 \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right) \left(\frac{10^2\ {\rm GeV}}{m_{\rm nlsp}}\right)^{\frac{1}{2}} < \Delta < 55 \left(\frac{f_a}{10^{12}\ {\rm GeV}}\right)^{\frac{2}{3}} \left(\frac{4\ {\rm MeV}}{T_{\rm min}^{\rm after}}\right)^{\frac{1}{3}} . \tag{39}$$

The lower bound on Δ shows that (36) is always stronger than (31), since the inequality (37) is always true as long as significant entropy is produced and if this were not the case, (31) would not be considered at all.

If saxions are part of the particle spectrum, (39) shows two things. i) It is likely that saxions produce significant entropy in their decays. To avoid it, one would have to restrict the reheating temperature such that they never enter equilibrium, or to choose safe values for f_a and $m_{\rm sax}$, e.g. $m_{\rm sax} = 1$ TeV and $f_a = 10^{10}$ GeV. ii) The corresponding dilution factor is much smaller than the maximal value allowed by cosmology and preferred as a solution of the NLSP decay problem.

The dilution factor can be increased by a larger axion decay constant, which makes the saxion more weakly interacting. From (39) we see that we need $f_a \simeq 5.2 \times 10^{13}$ GeV to reach the maximum $\Delta \simeq 0.75 \times 10^3$ of (22). This increases the decoupling temperature (29) and thereby the reheating temperature required to have the saxions in thermal equilibrium. If they did not enter equilibrium, the abundance would be $Y_{\rm sax} \ll Y_{\rm sax}^{\rm eq}$, and the saxion would be useless for our purpose. From the requirement $T_{\rm R} > T_{\rm sax}^{\rm dcp}$ and (29) we derive the upper bound

$$f_a \lesssim 1.0 \times 10^{12}\ \text{GeV} \left(\frac{T_{\rm R}}{4 \times 10^{12}\ \text{GeV}}\right)^{\frac{1}{2}} \left(\frac{\alpha_{\rm s}}{0.03}\right)^{\frac{3}{2}} , \tag{40}$$

where $\alpha_{\rm s}(4 \times 10^{12}\ \text{GeV}) \simeq 0.03$. Already such a $T_{\rm R}$ corresponds—at least in the case of heavy gravitinos—to an allowed but relatively large dilution factor $\Delta \sim 10^3$, cf. (7) and (10). For the small Δs of (39) the situation becomes worse and is in fact inconsistent with itself.

To sum up, if thermally produced saxions are to deliver the desired entropy, we need a large axion decay constant. Then we also need a large reheating temperature to make the saxion enter thermal equilibrium. This results in an overproduction of gravitinos, if they are produced without entering equilibrium, so that the scenario is not viable.

On the other hand, if the gravitino is so light that it enters equilibrium after reheating, the relic gravitino density becomes independent of the reheating temperature as mentioned in Sec. 3. Moreover, there could be another saxion production mechanism, which is the alternative we will concentrate on in the next section.

Let us therefore continue discussing the requirements of Tab. 1, turning to the decay products of the saxion. Due to R-parity conservation, it must produce sparticles in pairs and thus cannot decay into single gravitinos. Besides, the decay into gravitino pairs is negligible, since it is suppressed by an additional factor of $M_{\rm Pl}^2$. Consequently, requirement vi) is satisfied without any effort.

To fulfill requirement v) the decay into any other sparticle pair must be kinematically forbidden, i.e. $m_{\rm sax} < 2\, m_{\rm nlsp}$. This is the case if the saxion is lighter or not much heavier than the gravitino. Given that one expects both the gravitino and the saxion mass to be of order m_{susy}, such a spectrum does not seem unlikely. It is understood that requirement v) does not apply

for a light gravitino produced in thermal equilibrium, since the NLSP decays early enough before BBN and does not overproduce gravitinos.

On the other hand, the saxion may decay as well into two axions with [98]

$$\Gamma_{\rm sax}^{aa} \simeq \frac{x^2 m_{\rm sax}^3}{32\pi f_a^2}\,, \tag{41}$$

where the self-coupling x can be of order 1. In that case the Universe would be filled with relativistic axions during the process of entropy production. This would change the effective number of neutrino species, which could spoil the success of BBN. This requires $x \ll 1$ and there are concrete models realizing this [99].

Up to now, we went through the requirements i)–vi) of Tab. 1. We do not see any incompatibilities between the saxion producing entropy and gravitino dark matter. Hence, vii) is fulfilled automatically as well. Facing viii) we have to take care of the unavoidable by-products.

Axion Due to the similar coupling strength the axion also enters equilibrium, if the saxion does. Then its thermal abundance is the same as the saxion abundance (30). Due to its tiny mass [73]

$$m_a \simeq 0.62\ \mathrm{meV} \left(\frac{10^{10}\ \mathrm{GeV}}{f_a}\right) \tag{42}$$

its thermal relic density is negligibly small. The axion decays harmlessly into two photons with a lifetime many orders of magnitude larger than the age of the Universe [73], while the gravitino cannot decay into axions due to R-parity.

Axions are also produced by vacuum misalignment, which leads to the bound $f_a \lesssim 10^{12}$ GeV to avoid overproduction [73]. However, for considerable entropy production by the saxion this bound no longer holds, since the Universe is dominated by the saxion at the onset of axion oscillations. Then the axion density is given by [100]

$$\Omega_a h^2 \simeq 0.21 \left(\frac{T_{\rm sax}^{\rm dec}}{4\ \mathrm{MeV}}\right)\left(\frac{f_a}{10^{15}\ \mathrm{GeV}}\right)^2. \tag{43}$$

If we require $\Omega_a/\Omega_{\rm DM} = r \ll 1$, we find

$$\left(\frac{f_a}{10^{14}\ \mathrm{GeV}}\right)^2 \lesssim \left(\frac{r}{0.02}\right)\left(\frac{4\ \mathrm{MeV}}{T_{\rm sax}^{\rm dec}}\right), \tag{44}$$

so that values of $f_a > 10^{12}$ GeV are indeed allowed. The bound from $\Omega_a \ll \Omega_{3/2} \simeq \Omega_{\rm DM}$ is self-consistently cured by the decaying saxion.

Altogether, there is no problem at all with the axion in our scenario.

Axino This is different for the axino $\widetilde{a}$. Due to SUSY the axino has couplings similar to the saxion couplings and thus the same decoupling temperature (29) as the saxion and a similar relic abundance. The natural mass range for the axino is $\mathcal{O}(\text{keV}) < m_{\widetilde{a}} < \mathcal{O}(m_{3/2}) \sim m_{\text{susy}}$ [101, 98]. We demand $m_{\widetilde{a}} > m_{3/2}$ to keep the gravitino as LSP. Then the mass range for the axino becomes similar to that of the saxion. In comparison to the saxion it has the opposite R-eigenvalue and thus must produce sparticles in its decays. With a light gravitino and axino NLSP, one would arrive at another NLSP decay problem. The situation would be worse than our starting point. If the axino should not produce gravitinos, which would lead to $Y_{3/2} \sim Y_{\widetilde{a}}^{\text{eq}} = Y_{3/2}^{\text{eq}}$, there must be another decay channel kinematically open, i.e. $m_{\widetilde{a}} > m_{\text{nlsp}}$. In the most interesting case m_{nlsp} is close to the gravitino mass and the axino fulfills requirement vi).

If kinematically allowed, the axino is expected to decay dominantly into a gluino-gluon pair with

$$\Gamma_{\widetilde{a}}^{\widetilde{g}g} \simeq \frac{\alpha_{\text{s}}^2 m_{\widetilde{a}}^3}{128\pi^3 f_a^2} \,. \tag{45}$$

From the requirement of early enough decay we derive the same lower bound on $m_{\widetilde{a}}$ as found for the saxion (36). This would allow an axino lighter than the expected gluino mass. Weaker decays into sparticles and SM particles were to investigate. However, all these processes finally produce the lightest ordinary supersymmetric particle. The case of a heavy axino that decays after NLSP freeze-out has been studied in [102] for a neutralino dark matter scenario including the weaker decay into a neutralino and the re-annihilation of neutralinos. Even if the axinos are not in thermal equilibrium after inflation, they are regenerated by thermal scatterings and decays in the thermal plasma. The density produced in this way can be estimated in units of today's critical density as [103, 104, 105]

$$\Omega_{\widetilde{a}} h^2 \simeq 7.8 \times 10^2\, \Delta^{-1} \left(\frac{m_{\widetilde{a}}}{1\ \text{TeV}}\right) \left(\frac{T_{\text{R}}}{10^9\ \text{GeV}}\right) \left(\frac{10^{14}\ \text{GeV}}{f_a}\right)^2 . \tag{46}$$

The resulting neutralino density is many magnitudes larger than the thermal relic abundance (cf. e.g. Fig. 1), which in our scenario reintroduces the NLSP decay problem. In fact, the problem becomes much worse. Thus requirement v) is badly violated by the axino.

If we require the axino to decay before NLSP freeze-out, so that we do not have to care about the produced number of NLSPs since they thermalize normally, we find by a derivation analogous to that of (36) the lower bound on the axino mass

$$m_{\widetilde{a}} \gtrsim 1.2 \times 10^2\ \text{TeV} \left(\frac{m_{\text{nlsp}}}{10^2\ \text{GeV}}\right)^{\frac{2}{3}} \left(\frac{f_a}{10^{13}\ \text{GeV}}\right)^{\frac{2}{3}} \left(\frac{0.1}{\alpha_{\text{s}}}\right)^{\frac{2}{3}} \left(\frac{g_*(T_{\widetilde{a}}^{\text{dec}})}{100}\right)^{\frac{1}{6}} . \tag{47}$$

Since the gravitino problem could also be solved by making the gravitino comparably unnaturally heavy, such a large axino mass is not considered as a solution here. Furthermore, such an axino would produce considerable entropy with $\Delta_{\widetilde{a}} \simeq 290$. From the discussion at the end of Sec. 3.2 we know that this would spoil our scenario, since $\Delta_{\widetilde{a}}$ would dilute the saxion but not the NLSP. Thus, the situation is also inconsistent. The required axino mass to achieve $\Delta_{\widetilde{a}} = 1$ would be larger than about 10^7 TeV.

Altogether, requirement viii) of Tab. 1 is badly violated by the axino. Consequently, the thermally produced saxion—and obviously also the axino itself—is ruled out as viable particle to produce significant entropy after NLSP freeze-out. The exception to this conclusion is a light gravitino in thermal equilibrium after reheating, since it allows for high reheating temperatures and the NLSP decay problem is absent.

One may worry then if the strong CP problem can be solved by the Peccei-Quinn mechanism in scenarios of standard thermal leptogenesis with very light gravitino dark matter only. Going through the equations, especially from (47), we see that the axino becomes harmless for smaller axion decay constants $f_a \lesssim 10^{10}$ GeV with an acceptable axino mass $m_{\widetilde{a}} \gtrsim 1.2$ TeV. Since its decay into the gravitino is suppressed like $(f_a/M_{\mathrm{Pl}})^2$, the contribution to the gravitino density from axino decay is negligible. However, by inspection of (38) we see that in this case the saxion is unable to produce a significant amount of entropy. Then also the axion abundance restricts f_a to values smaller than about 10^{10} GeV.

In summary, by making the axino harmless we find that the thermally produced multiplet may also exist in scenarios of thermal leptogenesis with gravitino dark matter that does not enter equilibrium after reheating. However, the axion decay constant is restricted to a small window. Moreover, the thermally produced multiplet is in fact useless for our purpose. This is due to two generic features of the considered scenario: i) Superpartners have similar couplings and masses. ii) The same coupling—or at least couplings of the same strength—were responsible for production and late decay of the entropy-producing particle.

5.2.2 Generic Thermally Produced Particle

The negative result for the saxion can be generalized to other late-decaying particles that are produced in thermal equlibrium by processes controlled by the same coupling as the decay. As the simplest estimate, let us assume that the particle ϕ under consideration couples to SM particles via non-renormalisable interactions suppressed by an energy scale Λ and that the rate of reactions keeping ϕ in thermal equilibrium at high temperatures can be written as

$$\Gamma_\phi^{\mathrm{prod}} = x\,\frac{T^3}{\Lambda^2}\,, \tag{48}$$

where x is a model-dependent, dimensionless quantity containing couplings and kinematical factors, for example. The freeze-out from thermal equilibrium occurs for $H \simeq \Gamma_\phi^{\mathrm{prod}}$, which yields the decoupling temperature

$$T_\phi^{\mathrm{dcp}} \simeq \left(\frac{\pi^2 g_*(T_\phi^{\mathrm{dcp}})}{90}\right)^{\frac{1}{2}} \frac{\Lambda^2}{x M_{\mathrm{Pl}}} \simeq \frac{2.1\,\Lambda^2}{x \times 10^{18}\ \mathrm{GeV}}\,. \tag{49}$$

For the decay we estimate

$$\Gamma_\phi^{\mathrm{dec}} = y\,\frac{m_\phi^3}{\Lambda^2}\,, \tag{50}$$

where y contains model-dependent factors. Generically, we expect $x \lesssim y$, where kinematical factors and the relation between number density and temperature tend to lead to a somewhat smaller x. For instance, for the saxion we find $x \simeq 6 \times 10^{-7}$ and $y \simeq 3 \times 10^{-6}$. We obtain the temperature after the decay as discussed in Sec. 3.2,

$$T_\phi^{\mathrm{dec}} \simeq \left(\frac{45}{2\pi^2 g_*(T_\phi^{\mathrm{dec}})}\right)^{\frac{1}{4}} \frac{(y m_\phi^3 M_{\mathrm{Pl}})^{\frac{1}{2}}}{\Lambda} \simeq 1.1 \times 10^9\,\frac{y^{\frac{1}{2}} m_\phi^{\frac{3}{2}}\ \mathrm{GeV}^{\frac{1}{2}}}{\Lambda}\,, \tag{51}$$

assuming a sufficiently late decay to yield $g_*(T_\phi^{\mathrm{dec}}) = 10.75$. Together with the analogon of (32), which holds for any thermally produced scalar, and (17), we find the dilution factor

$$\Delta \simeq 1.1 \times 10^{-12}\,\frac{\Lambda}{(y m_\phi\,\mathrm{GeV})^{\frac{1}{2}}}\,, \tag{52}$$

estimating as before $\langle g_*^{1/3}\rangle \simeq g_*^{1/3}(T_\phi^{\mathrm{dec}})$.

Now we can use (7), (49), $\Omega_{3/2}^{\mathrm{tp}} \leq \Omega_{\mathrm{DM}}$ and $T_{\mathrm{R}} > T_\phi^{\mathrm{dcp}}$ to obtain a lower limit on Δ and thus a constraint on the model parameters,

$$\Delta \gtrsim \frac{6.8}{x}\left(\frac{\Lambda}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{M_{\tilde{g}}(m_{\mathrm{Z}})}{10^3\ \mathrm{GeV}}\right)^2 \left(\frac{100\ \mathrm{GeV}}{m_{3/2}}\right). \tag{53}$$

Furthermore, (17), (21) and (32) yield an upper limit on Δ, which can be combined with (53), resulting in

$$\frac{\Lambda}{(x m_\phi)^{\frac{1}{2}}} \lesssim 2.1 \times 10^{13}\ \mathrm{GeV}^{\frac{1}{2}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\tilde{g}}(m_{\mathrm{Z}})}\right)\left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{1}{2}}. \tag{54}$$

Plugging this bound into (52) yields the maximal dilution factor that can be realized with a thermally produced generic scalar,

$$\Delta \lesssim 24 \left(\frac{x}{y}\right)^{\frac{1}{2}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\tilde{g}}(m_{\mathrm{Z}})}\right)\left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{1}{2}}. \tag{55}$$

Using further combinations of (21) and (51)–(53), we find that this maximal dilution is reached for

$$\Lambda \simeq 1.9 \times 10^{14}\ \mathrm{GeV}\, \frac{x^{\frac{3}{4}}}{y^{\frac{1}{4}}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\widetilde{g}}(m_{\mathrm{Z}})}\right)^{\frac{3}{2}} \left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{3}{4}}, \tag{56}$$

$$m_\phi \simeq 79\ \mathrm{GeV} \left(\frac{x}{y}\right)^{\frac{1}{2}} \left(\frac{10^3\ \mathrm{GeV}}{M_{\widetilde{g}}(m_{\mathrm{Z}})}\right) \left(\frac{m_{3/2}}{100\ \mathrm{GeV}}\right)^{\frac{1}{2}}. \tag{57}$$

Thus, we conclude that the generalized scenario allows for the production of some entropy, but we do not expect a dilution factor large enough to solve the NLSP decay problem. In order to avoid this conclusion, we have to consider a situation where the mechanisms for production and decay are different, so that the decoupling temperature and the decay temperature are no longer connected.

5.2.3 Φ_{sax} as Oscillating Scalar

As the saxion corresponds to a flat direction of the scalar potential lifted by SUSY breaking effects, it can develop a large field value during inflation. It begins to oscillate around the potential minimum when the Hubble parameter becomes comparable to the saxion mass. This corresponds to the production of non-relativistic particles. The temperature at the onset of oscillations is

$$T_{\mathrm{sax}}^{\mathrm{osc}} \simeq 2.2 \times 10^{10}\ \mathrm{GeV} \left(\frac{m_{\mathrm{sax}}}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}} \left(\frac{228.75}{g_*(T_{\mathrm{sax}}^{\mathrm{osc}})}\right)^{\frac{1}{4}}. \tag{58}$$

Since we consider reheating temperatures higher than $T_{\mathrm{sax}}^{\mathrm{osc}}$ to enable thermal leptogenesis, the produced saxion abundance is independent of T_{R} and given by [106]

$$\begin{aligned} \frac{\rho_{\mathrm{sax}}^{\mathrm{osc}}}{s} &= \frac{1}{8} T_{\mathrm{sax}}^{\mathrm{osc}} \left(\frac{\phi_{\mathrm{sax}}^{\mathrm{i}}}{M_{\mathrm{Pl}}}\right)^2 \\ &\simeq 4.8\ \mathrm{GeV} \left(\frac{m_{\mathrm{sax}}}{1\ \mathrm{TeV}}\right)^{\frac{1}{2}} \left(\frac{f_a}{10^{14}\ \mathrm{GeV}}\right)^2 \left(\frac{\phi_{\mathrm{sax}}^{\mathrm{i}}}{f_a}\right)^2 \left(\frac{228.75}{g_*(T_{\mathrm{sax}}^{\mathrm{osc}})}\right)^{\frac{1}{4}}, \end{aligned} \tag{59}$$

where $\phi_{\mathrm{sax}}^{\mathrm{i}}$ denotes the initial amplitude of the oscillations and where we have assumed the simplest saxion potential, $V = \frac{1}{2} m_{\mathrm{sax}}^2 \phi_{\mathrm{sax}}^2$.

In this way production and decay are disconnected, circumventing the second feature mentioned at the end of the previous section. There is an additional free parameter, $\phi_{\mathrm{sax}}^{\mathrm{i}}$. The saxion density is constrained by requirement iv), i.e. that it should not dominate before NLSP freeze-out. For the limiting case of domination onset at $T_{\mathrm{nlsp}}^{\mathrm{fo}}$ we obtain from (15)

$$\frac{\rho_{\mathrm{sax}}^{\mathrm{osc}}}{s} = \frac{3}{4} T_{\mathrm{sax}}^{=} = \frac{3}{4} T_{\mathrm{nlsp}}^{\mathrm{fo}}. \tag{60}$$

Equalizing (59) and (60) we find for the initial amplitude

$$\left(\frac{\phi^{\mathrm{i}}_{\mathrm{sax}}}{M_{\mathrm{Pl}}}\right)^2 = 6\,\frac{T^{=}_{\mathrm{sax}}}{T^{\mathrm{osc}}_{\mathrm{sax}}} \tag{61}$$

or equivalently with $T^{=}_{\mathrm{sax}} = T^{\mathrm{fo}}_{\mathrm{nlsp}} \simeq m_{\mathrm{nlsp}}/25$

$$\left(\frac{\phi^{\mathrm{i}}_{\mathrm{sax}}}{f_a}\right) \simeq 2.6\times 10^4 \left(\frac{10^{10}\ \mathrm{GeV}}{f_a}\right)\left(\frac{8.4\ \mathrm{GeV}}{m_{\mathrm{sax}}}\right)^{\frac{1}{4}}\left(\frac{m_{\mathrm{nlsp}}}{10^2\ \mathrm{GeV}}\right)^{\frac{1}{2}}\left(\frac{g_*(T^{\mathrm{osc}}_{\mathrm{sax}})}{228.75}\right)^{\frac{1}{8}}. \tag{62}$$

The easiest expectation for the initial amplitude is $\phi^{\mathrm{i}}_{\mathrm{sax}} \sim M_{\mathrm{Pl}}$ or $\phi^{\mathrm{i}}_{\mathrm{sax}} \sim f_a$. Interestingly, the estimate (62) yields an initial amplitude $f_a < \phi^{\mathrm{i}}_{\mathrm{sax}} \sim \sqrt{f_a M_{\mathrm{Pl}}} < M_{\mathrm{Pl}}$, if we choose the harmless value $f_a = 10^{10}$ GeV found above. According to (17) and (35), the maximal dilution (22) is achieved for a saxion mass $m_{\mathrm{sax}} = 8.4$ GeV on the lower boundary from early enough decay (36). The axion multiplet comes into thermal equilibrium after reheating, which gives the known limit $m_{\widetilde{a}} \gtrsim 1.2$ TeV (47), avoiding problems with the axino. Thus, we have identified a working scenario where $\rho^{\mathrm{osc}}_{\mathrm{sax}} \gg \rho^{\mathrm{eq}}_{\mathrm{sax}}$, which enables significant entropy production while satisfying all requirements.

Smaller f_a are possible, too, provided that they respect the experimental bound (28). Larger f_a and $\phi^{\mathrm{i}}_{\mathrm{sax}}$ were not only in conflict with the scenario presented but also with standard cosmology. Furthermore, larger m_{sax} are allowed, while they lead following (35) to smaller Δs. From the naturalness point of view, the required small saxion mass—compared to $m_{\widetilde{a}}$ and m_{susy}—for maximal Δ might be the biggest concern. Nevertheless, we can conclude that the saxion as oscillating scalar can produce the desired entropy to soften the NLSP decay problem without violating any constraint from cosmology or observations.

We would like to stress that our scenario does not contain more requirements than the standard scenario with axion multiplet but no entropy production. Instead, we only have to change the allowed windows for some parameters, most importantly $\phi^{\mathrm{i}}_{\mathrm{sax}}$ and m_{sax}. Avoiding axion overproduction by vacuum misalignment even becomes easier. Other restrictions, in particular those on x in (41) as well as on f_a and $m_{\widetilde{a}}$, are the same as in the standard scenario. Note also that the initial amplitude of the saxion oscillations would have to be restricted to small values around f_a, if one wanted to prevent entropy production. In other words, the classical field value of the saxion endangers the standard scenario.

6 Conclusions

We have discussed the possibility to solve the gravitino problem in scenarios with standard thermal leptogenesis and thus a high reheating temperature by late-time entropy production. Our setup has been stable gravitino dark

matter with a neutralino NLSP. We have estimated that thermal leptogenesis is compatible with entropy production diluting the baryon asymmetry as well as the LSP and NLSP relic densities by up to three to four orders of magnitude. This amount of dilution roughly coincides with the maximum amount obtainable for radiation domination at NLSP freeze-out.

For a gravitino LSP with a mass of 100 GeV, which allows for a reheating temperature suitable for thermal leptogenesis, we have shown that a neutralino NLSP which is not much heavier can be diluted sufficiently to be compatible with BBN, i.e. its decays do not cause changes of the primordial light element abundances that are excluded by observations. However, this is only possible if the lightest neutralino contains a large wino or Higgsino component, whereas a bino-like neutralino remains excluded.

We have discussed the general requirements on the particle producing the desired entropy and found that it is severely constrained. On the other hand, in some sense all these requirements either have to be fulfilled by long-lived particles anyway or are generically fulfilled. As a specific example, we have discussed the saxion from the axion multiplet. We have found that sufficient entropy production is not possible for a thermally produced saxion, where the same couplings are relevant for its production and its decay. This is due to two conflicting requirements: on the one hand sufficient production requires sufficiently strong couplings, while on the other hand sufficiently late decay requires weak couplings, where later decay corresponds to more entropy production. In the considered case, the allowed parameter ranges fail to overlap. Using simple estimates, we have generalized this negative conclusion to generic thermally produced particles. Furthermore, we have encountered severe problems with the saxion's superpartner, the axino.

As an alternative, we have considered saxion production in coherent oscillations, which is independent of the saxion coupling. In this case, a relatively light saxion with a mass around 10 GeV is indeed able to satisfy all requirements. Thus, if the Peccei-Quinn mechanism solves the strong CP problem, the potentially dangerous saxion decays can in fact turn out as a fortune, solving the a priori unrelated gravitino problem.

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